



MATHEMATICS _ PAPER - 1



SECTION 1 (Maximum Marks : 18)

- This section contains **SIX** (06) questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :
 Full marks : +3 If **ONLY** the correct option is chosen;
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
 Negative Marks : -1 In all other cases.

1. Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$ is
 (A) 0 (B) 8000 (C) 8080 (D) 16000

Ans. D

2. If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = |x|(x - \sin x)$, then which of the following statements is TRUE ?
 (A) f is one-one, but NOT onto (B) f is onto, but NOT one-one
 (C) f is BOTH one-one and onto (D) f is NEITHER one-one NOR onto

Ans. C

3. Let the functions $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = e^{x-1} - e^{-|x-1|} \text{ and } g(x) = \frac{1}{2} (e^{x-1} + e^{1-x}).$$

Then the area of the region in the first quadrant bounded by the curves $y = f(x)$, $y = g(x)$ and $x = 0$ is.

- (A) $(2 - \sqrt{3}) + \frac{1}{2}(e - e^{-1})$ (B) $(2 + \sqrt{3}) + \frac{1}{2}(e - e^{-1})$
 (C) $(2 - \sqrt{3}) + \frac{1}{2}(e + e^{-1})$ (D) $(2 + \sqrt{3}) + \frac{1}{2}(e + e^{-1})$

Ans. A

4. Let a, b and λ be positive real numbers. Suppose P is an end point of the latus rectum of the parabola $y^2 = 4\lambda x$, and suppose the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the point P . If the tangents to the parabola and the ellipse at the point P are perpendicular to each other, then the eccentricity of the ellipse is.

- (A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$

Ans. A

5. Let C_1 and C_2 be two biased coins such that the probabilities of getting head in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Suppose α is the number of heads that appear when C_1 is tossed twice, independently, and suppose β is the number of heads that appear when C_2 is tossed twice, independently. Then the probability that the roots of the quadratic polynomial $x^2 - \alpha x + \beta$ are real and equal, is
- (A) $\frac{40}{81}$ (B) $\frac{20}{81}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. B

6. Consider all rectangles lying in the region $\{(x, y) \in \mathbb{R} \times \mathbb{R} : 0 \leq x \leq \frac{\pi}{2} \text{ and } 0 \leq y \leq 2 \sin(2x)\}$ and having one side on the x-axis. The area of the rectangle which has the maximum perimeter among all such rectangles, is
- (A) $\frac{3\pi}{2}$ (B) π (C) $\frac{\pi}{2\sqrt{3}}$ (D) $\frac{\pi\sqrt{3}}{2}$

Ans. C

SECTION 2 (Maximum Marks : 24)

- This section contains **SIX** (06) questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four options(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full marks	: +4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -2	In all other cases.

7. Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^3 - x^2 + (x - 1) \sin x$ and let $g: \mathbb{R} \rightarrow \mathbb{R}$ be an arbitrary function. Let $fg: \mathbb{R} \rightarrow \mathbb{R}$ be the product function defined by $(fg)(x) = f(x)g(x)$. Then which of the following statements is/are TRUE ?
- (A) If g is continuous at $x = 1$, then fg is differentiable at $x = 1$
 (B) If fg is differentiable at $x = 1$, then g is continuous at $x = 1$
 (C) If g is differentiable at $x = 1$, then fg is differentiable at $x = 1$
 (D) If fg is differentiable at $x = 1$, then g is differentiable at $x = 1$

Ans. A,C

8. Let M be a 3×3 invertible matrix with real entries and let I denote the 3×3 identity matrix. If $M^{-1} = \text{adj}(\text{adj } M)$, then which of the following statements is/are ALWAYS TRUE ?
 (A) $M = I$ (B) $\det M = 1$ (C) $M^2 = I$ (D) $(\text{adj } M^2) = I$

Ans. B,C,D

9. Let S be the set of all complex numbers z satisfying $|z^2+z+1| = 1$. Then which of the following statements is/are TRUE ?

- (A) $\left|z + \frac{1}{2}\right| \leq \frac{1}{2}$ for all $z \in S$ (B) $|z| \leq 2$ for all $z \in S$
 (C) $\left|z + \frac{1}{2}\right| \geq \frac{1}{2}$ for all $z \in S$ (D) The set S has exactly four elements

Ans. B,C

10. Let x, y and z be positive real numbers. Suppose x, y and z are the lengths of the sides of a triangle opposite to its angles X, Y and Z , respectively. If $\tan \frac{X}{2} + \tan \frac{Z}{2} = \frac{2y}{x+y+z}$, then which of the following statements is/are TRUE ?

- (A) $2Y = X + Z$ (B) $Y = X + Z$ (C) $\tan \frac{X}{2} = \frac{x}{y+z}$ (D) $x^2 + z^2 - y^2 = xz$

Ans. B,C

11. Let L_1 and L_2 be the following straight lines.

$$L_1: \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{3} \text{ and } L_2: \frac{x-1}{-3} = \frac{y}{-1} = \frac{z-1}{1}$$

Suppose the straight line $L: \frac{x-\alpha}{l} = \frac{y-1}{m} = \frac{z-\gamma}{-2}$

lies in the plane containing L_1 and L_2 , and passes through the point of intersection of L_1 and L_2 .

If the line L bisects the acute angle between the lines L_1 and L_2 , then which of the following statements is/are TRUE?

- (A) $\alpha - \gamma = 3$ (B) $l+m=2$ (C) $\alpha - \gamma = 1$ (D) $l+m=0$

Ans. A,B

12. Which of the following inequalities is/are TRUE?

(A) $\int_0^1 x \cos x \, dx \geq \frac{3}{8}$

(B) $\int_0^1 x \sin x \, dx \geq \frac{3}{10}$

(C) $\int_0^1 x^2 \cos x \, dx \geq \frac{1}{2}$

(D) $\int_0^1 x^2 \sin x \, dx \geq \frac{2}{9}$

Ans. **A,B,D**

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round -off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme :

Full marks	: +4	If ONLY the correct numerical value is entered;
Zero Marks	: 0	In all other cases.

13. Let m be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is _____

Ans. **8.00**

14. Let a_1, a_2, a_3, \dots be a sequence of positive integers in arithmetic progression with common difference 2. Also, let b_1, b_2, b_3, \dots be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of c , for which the equality $2(a_1 + a_2 + \dots + a_n) = b_1 + b_2 + \dots + b_n$ holds for some positive integer n , is _____

Ans. **1.00**

15. Let $f: [0, 2] \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = (3 - \sin(2\pi x)) \sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$$

If $\alpha, \beta \in [0, 2]$ are such that $\{x \in [0, 2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is_____

Ans. 1.00

16. In a triangle PQR, let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\frac{\vec{a} \cdot (\vec{c} - \vec{b})}{\vec{c} \cdot (\vec{a} - \vec{b})} = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}$,

then the value of $|\vec{a} \times \vec{b}|^2$ is_____

Ans. 108.00

17. For a polynomial $g(x)$ with real coefficients, let m_g denote the number of distinct real roots of $g(x)$. Suppose S is the set of polynomials with real coefficients defined by

$$S = \left\{ (x^2 - 1)^2 (a_0 + a_1x + a_2x^2 + a_3x^3) : a_0, a_1, a_2, a_3 \in \mathbb{R} \right\}$$

For a polynomial f , let f' and f'' denote its first and second order derivatives, respectively. Then

the minimum possible value of $(m_{f'} + m_{f''})$, where $f \in S$, is_____

Ans. 5.00

18. Let e denote the base of the natural logarithm. The value of the real number a for which the right

hand limit $\lim_{x \rightarrow 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$ is equal to a nonzero real number, is_____

Ans. 1.00