Learning Temple
IIT/NEET ACADEMY

PHYSICS [JEE ADVANCED - 2019] PAPER - 1

Section -1 (Maximum Marks : 12)

- This section contains **Four (04)** question.
- Each question has **Four option ONLY ONE** of these four options is the correct answer.
- Each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme.
  Full Marks : +3 If only Y the correct option is chosen.
  Zero Marks : 0 If none of the option is chosen (i.e. the question is unanswered)
  Negative Marks : –1 in all other cases.

1. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as

$$T(t) = T_0(1 + \beta t^{1/4})$$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of of temperature. The heat capacity of the metal is :

(1) $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$ (2) $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$ (3) $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (4) $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$

Ans. **4**

dQ = HdT

$$\frac{dQ}{dt} = H.\frac{dT}{dt}$$

$$P = H.\ T_0\ .\ \beta.\ \frac{1}{4}\ .\ t^{-3/4}$$

$$\frac{4P}{T_0.\beta} = t^{-3/4}.H$$

Now $T - T_0 = T_0\beta t^{1/4}$

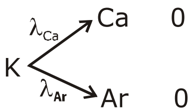
So $t^{3/4} = \left(\frac{T-T_0}{T_0\beta}\right)^3$

$\therefore$ $H = \frac{4P(T-T_0)^3}{T_0^4\beta^4}$

2. In a radioactive sample. ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time $t\times10^9$ years. If the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99. The value of t will be. [Given : In 10 = 2.3]

(1) 1.15 (2) 9.2 (3) 4.6 (4) 2.3

Sol. **2**



t = 0

$$\frac{dN}{dt} = -(\lambda_1 + \lambda_2) \times N$$

$$\log_e\left(\frac{N}{N_0}\right) = -(\lambda + \lambda_2)t$$

$$2.3 \times \log_{10}\left(\frac{N_0}{N_0/100}\right) = 5 \times 10^{-10}t$$

$$\frac{2.303 \times 2}{5 \times 10^{-10}} = t$$

$2.303 \times 0.4 \times 10^{10} = t$

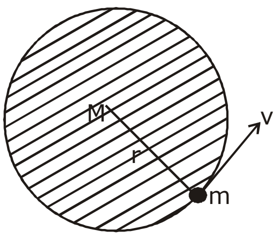
$t = 9.2 \times 10^{9}$ year

3. Consider a spherical gaseous cloud of mass density $\rho(r)$ in a free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant with time. the particle number density $n(r) = \rho(r)/m$ is : (G = universal gravitational constant)

(1) $\frac{K}{\pi r^2 m^2 G}$ (2) $\frac{3K}{\pi r^2 m^2 G}$ (3) $\frac{K}{2\pi r^2 m^2 G}$ (4) $\frac{K}{6\pi r^2 m^2 G}$

Ans. **3**

Sol.



$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$= \frac{2}{r}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad \frac{GMm}{r^2} = \frac{2K}{r}$$

$$\Rightarrow \quad M = \frac{2Kr}{Gm}$$

$\Rightarrow \quad dM = \frac{2K}{Gm}dr$

$\Rightarrow \quad 4\pi r^2 dr\rho = \frac{2K}{Gm}dr$

$\therefore \quad \rho = \frac{K}{2\pi Gmr^2}$

$\Rightarrow \frac{p}{m} = \frac{k}{2\pi Gm^2r^2}$

4. A thin spherical insulating shell of radius R caries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2$ ($\alpha << 1$) is made on the shell without affecting the rest of the shell. Which one of the following statements is correct.

(1) The ratio of potential at the center of the shell to that of the point at $\frac{1}{2}$ R from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

(2) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

(3) The magnitude of electric field at a point located on a line passing through the hole and shell's center on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

(4) The potential at the center of shell is reduced by $2\alpha V_0$.

Sol. **1**

$dq = \frac{Q}{4\pi R^2}dA = Q\alpha$

Given

V at surface

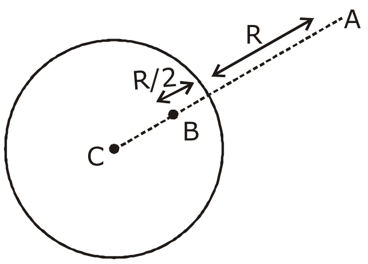
$V_0 = \frac{KQ}{R}$

V at C

$V_C = \frac{KQ}{R} - \frac{K\alpha Q}{R} = V_0(1-\alpha)$

V at B

$V_B = \frac{KQ}{R} - \frac{K(\alpha Q)}{R/2} = V_0(1-2\alpha)$



$\therefore \quad \frac{V_C}{V_B} = \frac{1-\alpha}{1-2\alpha}$ (Option 1)

E at A

$E_A = \frac{KQ}{(2R)^2} - \frac{K\alpha Q}{R^2} = \frac{KQ}{4R^2} - \frac{\alpha V_0}{R}$

So reduced by $\frac{\alpha V_0}{R}$

E at C

$E_C = \frac{K(\alpha Q)}{R^2} = \frac{\alpha V_0}{R}$

So increased by $\frac{\alpha V_0}{R}$

Section -2 (Maximum Marks : 32)

- This section contains **Eight (08)** question.
- Each question has **Four** options **ONE OR MORE THAN ONE** of these four options is(are) correct answers.
- For each question, choose the option(s) corresponding to (all) the correct answers.
- Answer to each question will be evaluated according to the following marking scheme.
  Full Marks : +4 If only (all) the correct option(s) is (are) is chosen.
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If none of the options is chosed (i.e. the question is unanswered).
  Negative Marks : –1 in all other cases.
- For example in a questions, If (A), (B) and (D) are the ONLY three options corresponding to correct answer, then
  Choosing ONLY (A), (B) and (D) will get +4 marks.
  Choosing ONLY (A) and (B) will get +2 marks;
  Choosing ONLY (A) and (D) will get +2 marks.
  Choosing ONLY (B) and (D) will get +2 marks;
  Choosing ONLY (A) will get +1 marks;
  Choosing ONLY (B) will get +1 marks;
  Choosing ONLY (D) will get +1 marks;
  Choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark.

1. Two identical moving coil galvanometers have 10Ω resistance and full scale deflection at $2\mu A$ current. One of them is converted into a voltmeter of 100 m V full scale reading and the other into an Ammeter of 1mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct?

(1) The resistance of the Ammeter will be 0.02Ω (round off to 2nd decimal place)

(2) The measured value of R will be $978\ \Omega < R < 982\ \Omega$

(3) If the ideal cell is replaced by a cell having internal resistance of 5Ω then the measured value of R will be more than 1000Ω

(4) The resistance of the voltmeter will be $100\ k\Omega$

Sol. **1,2**

$r_g = 10\ \Omega \quad i_g = 2\mu A$

For volt meter $v = ig\,(r_g + S)$

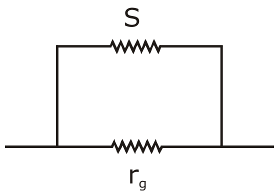
$100 \times 10^{-3} = 2 \times 10^{-6}\ \underbrace{10 + S}_{R_V}$

$R_v = r_q + S$

$R_V = 50000\ \Omega$ (Option 4 is incorrect)

For ammeter $i = i_g\left(1 + \frac{r_g}{S}\right)$ $\qquad R_A = \frac{r_g \times S}{r_g + S}$

$i = i_g\left(\frac{r_g + S}{S}\right)$ $\qquad \frac{R_A}{r_g} = \frac{S}{r_g + S}$

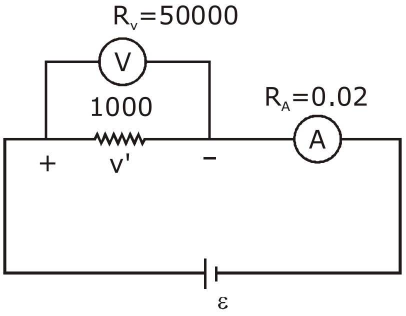


$i = i_g \frac{r_g}{R_A}$

$R_A = 1 \times 10^{-3} = 2 \times 10^{-6} \times 10$

$R_A = 2 \times 10^{-2}$

$= 0.02$ option (1)



$V' = \varepsilon - iR_A$

$\frac{\varepsilon}{I} = R_\varepsilon n = \frac{50000 \times 1000}{51000} + 0.02$

∴ Option (2)

∴ resistance measured

= v'/i

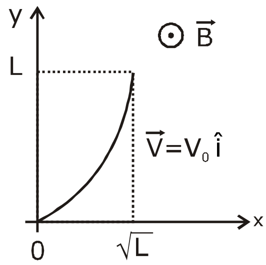
= ε-iR_A/i

= ε/i - R_A

= 50000/51 = 980.342

Internal resistance will not change any their in otpion (2)

2. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0 L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are :



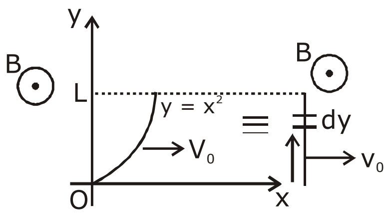
(1) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$

(2) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$

(3) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

(4) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

Sol. **1,3,4**



For calculating the motional emf across the length of the wire, let us project wire such that $\vec{B}, \vec{v}, \hat{\ell}$ becomes mutually orthogonal. Thus

$$d\varepsilon = Bv_0\,dy = B_0\left[1+\left(\frac{y}{L}\right)^{\beta}\right]V_0dy$$

$$\varepsilon = \int_0^L B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)V_0dy$$

$$= B_0V_0L\left[1+\frac{1}{\beta+1}\right]$$

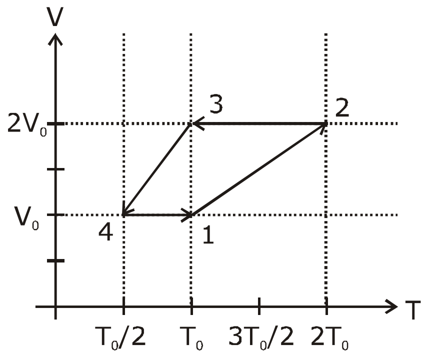
emf in loop is proportional to L for given value of β.

for
$\beta = 0$; $\varepsilon = 2B_0V_0L$

$\beta = 0$; $\varepsilon = B_0V_0L\left[1+\frac{1}{3}\right] = \frac{4}{3}B_0V_0L$

the length of the projection of the wire y = x of length $\sqrt{2}L$ on the y-axis is L thus the answer remain unchanged

3. One mole of a monatomic ideal gas goes through a thermodynamic cucle, as shown in the volume versus temperature (V–T) diagram. The correct statement(s) is/are:
[R is the gas constant]



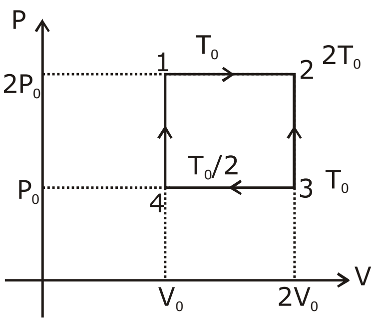
(1) The ratio of heat transfer during processes 1→2 and 2→3 is $\left|\frac{Q_{1\to2}}{Q_{2\to2}}\right| = \frac{5}{3}$

(2) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(3) Work done in this thermodynamic cycle (1→2→3→4→1) is $|W| = \frac{1}{2}RT_0$

(4) The ratio of heat transfer during processes 1→2 and 3→4 is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

Sol. **1,3**



(A) $\left|\dfrac{\Delta Q_{1\to2}}{\Delta Q_{3\to4}}\right| \left|\dfrac{NC_p\Delta T_{1\to2}}{NC_p\Delta T_{3\to4}}\right| = \dfrac{T_0}{T_0/2} = 2$

(B) $\left|\dfrac{\Delta Q_{1\to2}}{\Delta Q_{2\to3}}\right| = \left|\dfrac{NC_p\Delta T_{1\to2}}{NC_v\Delta T_{2\to3}}\right| = \dfrac{C_P}{C_V} = \dfrac{5}{3}$

(C) $W_{cycle} = P_0V_0 = nR\left[\dfrac{T_0}{2}\right]$ (Using point no. 4)

(D) wrong as no adiabatic process is involved

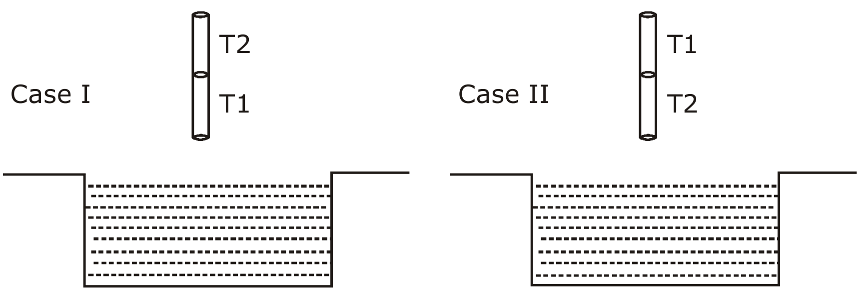
4. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct ?

(1) The dimension of force is L^{-3}
(2) The dimension of energy is L^{-2}
(3) The dimension of power is L^{-5}
(4) The dimension of linear momentum is L^{-1}

Sol. **1,2,4**

$[M] = [\text{Mass}] = [M^0L^0T^0]$
$[J] = [\text{Angular momentum}] = [ML^2T^{-1}]$
$[L] = [\text{Length}]$
Now ; $[ML^2T^{-1}] = [M^0L^0T^0]$
$\therefore \quad [L^2] = [T]$
Power $[P] = [MLT^{-2}.\ LT^{-1}]$
$= [ML^2T^{-3}]$
$= [L^2L^{-6}]$
$[P] = [L^{-4}]$
Energy/work $[W] = [MLT^{-2}.L]$
$= [L^2L^{-4}]$
$= [L^{-2}]$
Force $[F] = [MLT^{-2}] = [L\ .\ L^{-4}] = [L^{-3}]$
Linear momentum $[p] = [MLT^{-1}] = [L.L^{-2}]$
$[p] = [L^{-1}]$

5. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is(are) correct ?
[Surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2]



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(1) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.
(2) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)
(3) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)
(4) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

Ans. 1,2,4 or 2,4

Balancing length in T_1

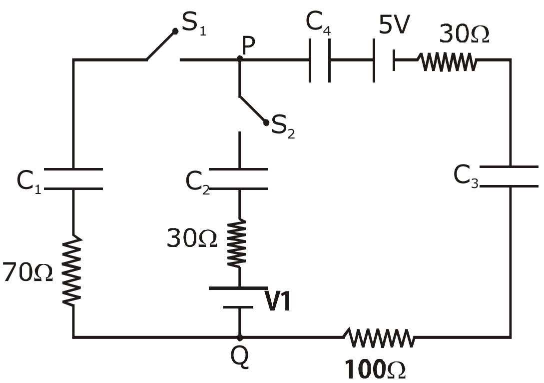
$$h = \frac{2(0.075)\cos 0^\circ}{R\rho g} = 7.5 \text{ cm}$$

Balancing length in T_2

$$h = \frac{2(0.075)\cos 60^\circ}{R\rho g} = 3.75 \text{ cm}$$

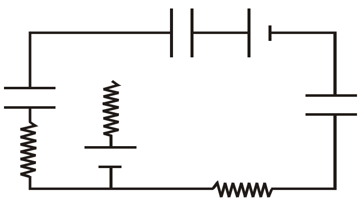
(iii) If $(l)_{\tau_1}$ < 7.5 cm then meniscus will adjust its radius of curvature according to the situation but water will not enter in to t_2 option 1 may or may not be correct its depends upon the situation.
Ans. is either 1, 2, 4 or 2, 4

6. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are C_1 = 10 μF, C_2 = 30 μF and $C_3 = C_4$ = 80 μF.



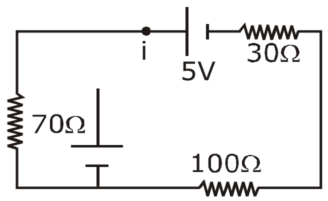
Which of the statement(s) is/are correct ?
(1) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A(round off to 1st decimal place).
(2) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitor C_1 will be 4V.
(3) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage, difference between points P and Q will be 10 V.
(4) At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.

Sol. **2,4**



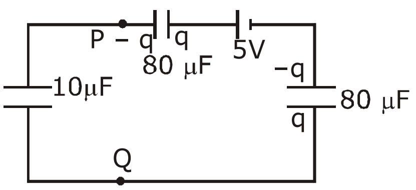
Just after closing of switch charge on C is zero.

$\therefore$ Replace all capacitors with wire.



$$i = \frac{5}{70+100+30} = \frac{5}{200} = 25mA$$

Now S_1 is kept closed for long time circuit is in steady state



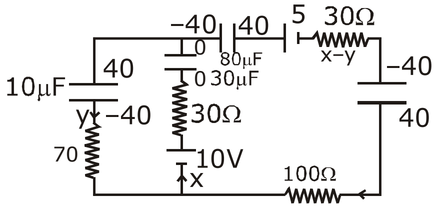
$$\frac{9}{10} + \frac{9}{80} + \frac{9}{80} - 5 = 0$$

$$\frac{109}{80} = 5$$

$\therefore$ q = 40 μC

$\therefore$ V across C_1 = 40/10 = 4 volt

Now just after closing of S_2 charge on each capacitor remain same



KVL

– 10 +x × 30 + 40/10 + y × 70 = 0

30x + 70y = 6 ... (1)

$$\frac{40}{80} + 5 + (x-y)30 - \frac{40}{80} + (x-y)\times 100 - 10 + x\times 30 = 0$$

160x – 130y – 6 = 0.... (2)

y = 96/1510

x = 0.05 amp.

7. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r & with its center same as that of the shell. Here, center of cylinder is a point on the axis of the cylinder which is equidistant from its top & bottom surfaces. Which of the following option (s) is/are correct ?
[ε_0 is the permittivity of free space]

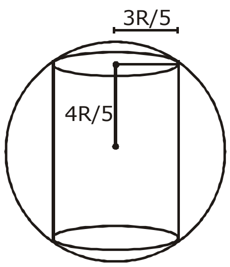
(1) If $h < \frac{8R}{5}$ and $r = \frac{3R}{5}$ then $\phi = 0$

(2) If h > 2R and r = $\frac{3R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

(3) If h > 2R and r > R then $\phi = \frac{Q}{\varepsilon_0}$

(3) if h > 2R and r = $\frac{4R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

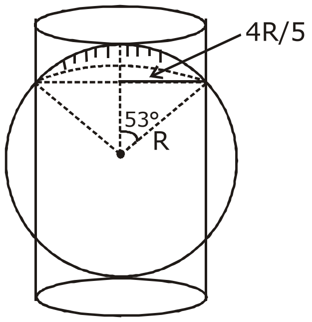
Sol. **1,2,3**



$\phi = 0$

so for $h < \frac{8R}{5}$ $\phi = 0$

(C) for h = 2R $r = \frac{4R}{5}$



Shaded charge = $2\pi(1 - \cos 53°) \times \frac{Q}{4\pi}$

$\therefore \quad = \frac{Q}{5}$

$\therefore \quad q_{enclosed} = \frac{2Q}{5}$

$\therefore \quad \phi = \frac{2Q}{5\varepsilon_0}$

$\therefore \quad$ for $h > 2R$ $r = \frac{4R}{5}$

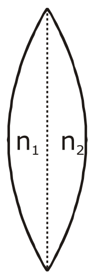
$\therefore \quad \phi = \frac{2Q}{5\varepsilon_0}$

(d) line option C for $h = 2R$ $\; r = \frac{3R}{5}$

$q_{enclosed} = 2 \times 2\pi(1 - \cos 37°)\ \frac{Q}{4\pi} = \frac{Q}{5}$

$\therefore \quad \phi = \frac{Q}{5\varepsilon_0}$

8. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n - 1)$ and $1 < n < 2$, the correct statement(s) is/are.



(1) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

(2) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(3) The relation between $\frac{\Delta f}{f}$ and $\left|\frac{\Delta n}{n}\right|$ remains unchanged if both the convex surfaces are re-placed by concave surfaces of the same radius of curvature.

(4) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place).

Sol. **1,3,4**

When $n_1 = n_2 = n$

$\frac{1}{f} = (n - 1) \times \frac{2}{R}$

So, $f = \dfrac{R}{2(n-1)}$ ....(i)

2nd Case:

$$\frac{1}{f_1} = \frac{n-1}{R}$$

$$\frac{1}{f_2} = \frac{(n+\Delta n)-1}{R}$$

$$\frac{1}{f_{eq}} = \frac{1}{f+\Delta f} = \left(\frac{n-1}{R}\right) + \frac{(n+\Delta n)-1}{R} = \frac{2(n-1)+\Delta n}{R}$$

$$\Delta f = \left(\frac{R}{2(n-1)+\Delta n}\right) - \left(\frac{R}{2(n-1)}\right)$$

$$= \frac{R}{2}\left[\frac{(n-1)-(n-1+\Delta n)}{(n-1+\Delta n)(n-1)}\right] = \frac{-\Delta n}{(n-1)^2} \times \frac{R}{2}$$

$$\frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)} \qquad ...(2)$$

(1) If $\dfrac{\Delta n}{n} < 0$ then $\dfrac{\Delta f}{f} > 0$ from equation (1)

(2) $2n - 2 < n$ because $n < 2$

$$\Rightarrow \frac{\Delta f}{f} = \frac{1}{2}\left|\frac{\Delta n}{n-1}\right| > \frac{\Delta n}{n}$$

So, $\dfrac{\Delta f}{f} > \left|\dfrac{\Delta n}{n}\right|$ So (2) is wrong

(3) Relation between $\dfrac{\Delta f}{f}$ and $\dfrac{\Delta n}{n}$ is independent of R so (3) is correct

(4) $|\Delta f| = \dfrac{f\Delta n}{(n-1)} = \dfrac{(20\times10^{-3})}{1.5-1} = 40\times10^{-3} = 0.04$

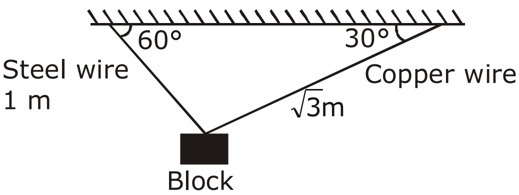
SECTION - 3 [MAXIMUM MARKS : 18]

This section contains six (06) questions. The answer to each question is a Numerical value.
For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numerical keypad in the place designated to enter teh answer. if the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
Answer to each question will be evaluated according to the following marking scheme.

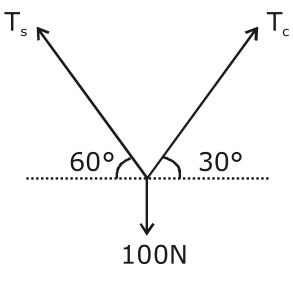
Full Marks : +3 If ONLY the correct numerical value is entered
Zero Marks : 0 in all other cases.

1. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is (Δl_C) and elongation in steel wire is (Δl_s), then the ratio $\frac{\Delta l_C}{\Delta l_S}$ is -

[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2, respectively]



Sol. **2**



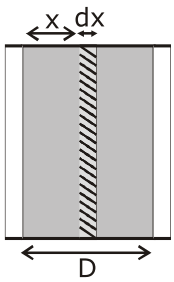
$$\frac{T_s}{2} = T_c \frac{\sqrt{3}}{2}$$

$$T_s = \sqrt{3} T_c$$

$$\frac{\Delta \ell_c}{\Delta \ell_s} = \left(\frac{T_c}{T_s}\right)\left(\frac{\ell_C}{\ell_S}\right)\left(\frac{Y_S}{Y_C}\right) = \left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{1}\right)\left(\frac{2 \times 10^{11}}{1 \times 10^{11}}\right) = 2$$

2. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N (> 10^3), the capacitance C is $\alpha\left(\frac{k\varepsilon_0 A}{d\ln 2}\right)$. The value of α will be -
[ε_0 is the permittivity of free space]

Sol. **1**



$$\frac{x}{m} = \frac{D}{N}$$

$$d\left(\frac{1}{C}\right) = \frac{dx}{K_m\varepsilon_0 A} = \frac{dx}{K\varepsilon_0 A\left(1+\frac{m}{N}\right)} = \frac{dx}{K\varepsilon_0 A\left(1+\frac{X}{D}\right)}$$

$$\frac{1}{C_{eq}} = \int d\left(\frac{1}{C}\right) = \int_0^D \frac{Ddx}{K\varepsilon_0 A(D+x)}$$

$$\frac{1}{C_{eq}} = \frac{D}{K\varepsilon_0 A}\ln 2$$

$$C_{eq} = \frac{K\varepsilon_0 A}{D\ln 2}$$. therefore $\alpha = 1$

3. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boilding temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be ____C°.
[Neglect the heat exchange with surrounding.

Sol. **270**

Let m = mass of calorimeter,
x = specific heat of calorimeter
s = specific heat of liquid
L = latent heat of liquid

First 5 g of liquid at 30° is poured to calorimeter at 110°C

$\therefore$ m × x × (100 – 80) = 5 × s × (80 × 30) + 5 L

$\Rightarrow$ mx × 30 = 250 s + 5 L ....(i)

Now, 80 g of liquid at 30° is poured into calorimeter at 80°C, the equilibrium temperature reaches to 50°C

$\therefore$ m × x × (80 – 30) = 80 × s × (50 – 30)

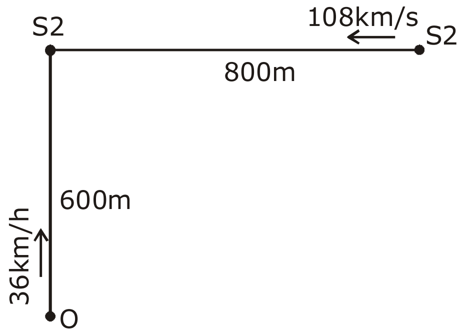
$\Rightarrow$ mx × 30 = 1600 s ....(ii)

From (i) and (ii)

250 s + 5 L = 1600 s $\Rightarrow$ 5L = 1350 s

$\Rightarrow \frac{L}{s} = 270$

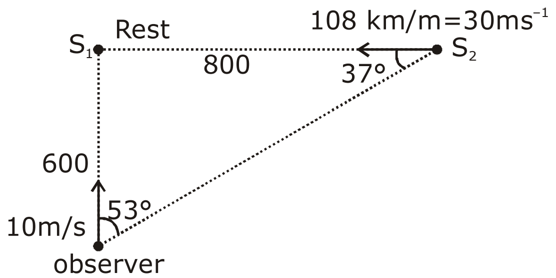
4. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is ________. [Speed of the sound = 330 m/s]



Sol. **8.12 to 8.13**

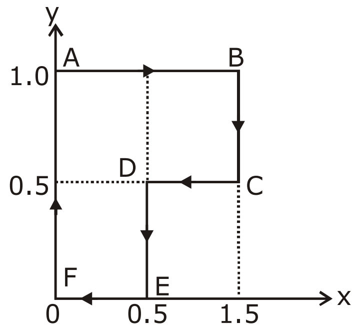
Speed of sound = 330 m/s

Calculate beat f_{req}



$$f_b = 120\left[\left(\frac{330+10\cos 53^\circ}{330-30\cos 37^\circ}\right)-\left[\frac{330+10}{330}\right]\right] = 120\left[\frac{336}{306}-\frac{34}{33}\right] = 8.128 \text{ Hz}$$

5. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (ay\hat{L} + 2ax\hat{j})$ N, Where x and y are in meter and α =-1 Nm^{-1}. The work done on the particle by this force $\vec{F}$ will be __ Joule.



Sol. **0.75 J**

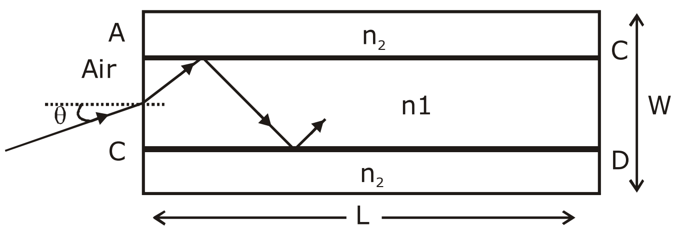
As α = −1

$$\therefore\ \vec{F} = \underbrace{-y\hat{i} - x\hat{j}}_{1} - x\hat{j}$$

This is now a perfect differential format whose work done is zero for a complete cycle.
Hence for −xj only WD needs to be calculated.

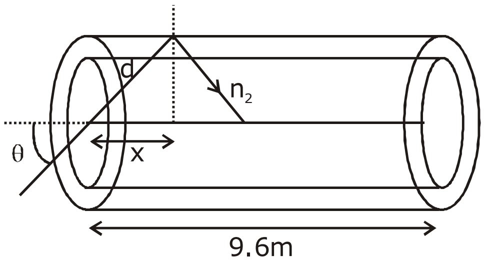
$\therefore$ W = 1×0.5 + 0.5 × 0.5
= 0.5 + 0.25
= 0.75 J

6. A planar structure of length *L* and width *W* is made of two different optical media of refractive indices n_1=1.5 and n_2=1.44 as shown in figure. If *L*>>*W*. a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L=9.6m, if the incident angle θ is varried, the maximum time taken by ray to exit the plane CD is t x 10^{-9} S, where t is ___
[Speed of light c=3x10^8 m/s]



Sol. 50

x = 5



$1.5 \sin \theta_0 = 1.44 \sin 90°$

$\sin \theta_c = \frac{1.44}{1.50} = \frac{24}{25}$

$\therefore \sin \theta_c = \frac{x}{d} = \frac{24}{25}$

$d = \frac{25x}{24}$

$\therefore$ total length travel by light $= \frac{25}{24} \times 9.6 = 10$ m

$\therefore t = \frac{S}{\left(\frac{C}{n_2}\right)} = \frac{10}{\frac{3\times10^8}{1.5}}$

$= \frac{1}{2} \times 10^{-7} = 5 \times 10^{-8}$

t = 50 ns

$t = 5 \times 10^{-8}$