Learning Temple
IIT/NEET ACADEMY

MATHS [JEE ADVANCED - 2019] PAPER - 2

SECTION–1 (Maximum marks :32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.
  Full marks : +4 If only (all) the correct option(s) is (are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen and both of which are correct
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If two or more options is chosen (i.e. the question is unanswered)
  Negative Marks : –1 in all other cases
- For example, in a question, if (A),(B) and (D) are the ONLY three options corresponding to correct answer, then
  choosing ONLY (A), (B) and (D) will get +4 marks
  choosing ONLY (A) and (B) will get +2 marks
  choosing ONLY (A) and (D) will get +2 marks
  choosing ONLY (B) and (D) will get +2 marks
  choosing ONLY (A) will get +1 mark
  choosing ONLY (B) will get +1 mark
  choosing ONLY (D) will get +1 mark
  choosing no option (i.e., the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark

1. Three lines

$L_1 : \vec{r} = \lambda\hat{i} \quad \lambda \in R$

$L_2 : \vec{r} = \hat{k} + \mu\hat{i}, \mu \in R$

$L_3 : \vec{r} = \hat{i} + \hat{j} + \nu\hat{k}, \ \nu \in R$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P,Q and R are collinear ?

(1) $\hat{k} + \hat{j}$ (2) $\hat{k} - \frac{1}{2}\hat{j}$ (3) $\hat{k}$ (4) $\hat{k} + \frac{1}{2}\hat{j}$

Sol. **2,4**

2. Let f : $R \to R$ be given by f(x) = (x–1)(x–2)(x–5). Define

$$F(x)=\int_0^x f(t)dt, \quad x>0$$

Then which of the following options is/are correct ?

(1) F has two local maxima and one local minimum in $(0,\infty)$
(2) F has a local maximum at x = 2
(3) $F(x) \neq 0$ for all $x \in (0,5)$
(4) F has a local minimum at x = 1

Sol. **2,3,4**

3. Let $f: R \to R$ be a function We say that f has

PROPERTY 1 if $\lim_{h\to 0}\frac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim_{h\to 0}\frac{f(h)-f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct ?

(1) $f(x) = |x|$ has PROPERTY 1

(2) $f(x) = x|x|$ has PROPERTY 2

(3) $f(x) = x^{2/3}$ has PROPERTY 1

(4) $f(x) = \sin x$ has PROPERTY 2

Sol. **1,3**

4. For non-negative integers n, let

$$f(n) = \frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0,\pi]$, which of the following options is/are correct ?

(1) $\sin(7\cos^{-1} f(5))=0$

(2) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2+2\alpha-1=0$

(3) $\lim_{n\to 0} f(n) = \frac{1}{2}$

(4) $f(4)=\frac{\sqrt{3}}{2}$

Sol. **1,2,4**

5. Let $x \in R$ and let

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \qquad Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ and } R = PQP^{-1}.$$

Then which of the following options is/are correct ?

(1) For x = 0, if $R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, then a+b = 5

(2) There exists a real number x such that PQ = QP

(3) For x = 1, there exists a unit vector $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ for which $R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(4) $\det R = \det\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$, for all $x \in R$

Sol. **1,4**

6. Let $f(x) = \frac{\sin \pi x}{x^2}$, $x > 0$.

Let $x_1 < x_2 < x_3 < \ldots < x_n < \ldots$ be all the points of local maximum of f and $y_1 < y_2 < y_3 < \ldots . < y_n < \ldots .$ be all the points of local minimum of f.
Then which of the following options is/are correct ?

(1) $x_1 < y_1$

(2) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n

(3) $| x_n - y_n | > 1$ for every n

(4) $x_{n+1} - x_n > 2$ for every n

Sol. **1,3,4**

7. Let $P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$,

and $X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_K^T$

Where P_K^T denotes the transpose of the matrix P_K. Then which of the following options is/are correct ?

(1) The sum of diagonal entries of X is 18

(2) If $X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$

(3) X is a symmetric matrix

(4) X – 30I is an invertible matrix

Sol. **1,2,3**

8. For

For $a \in R|a| > 1$, let

$$\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+\ldots+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+\ldots+\frac{1}{(an+n)^2}\right)}\right)=54.$$

Then the possible value(s) of a is/are

(1) 7 (2) –6 (3) 8 (4) –9

Sol. **3,4**

Section 2

- This section contains **SIX (06)** qeustions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme;
  Full Marks : +3 If ONLY the correct numerical value is entered
  Zero Marks : 0 in all other cases.

1. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. $\alpha, \beta, \in \mathbb{R}$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$ equals_____

Sol. **18**

2. Suppose

$$\det \begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^{n}C_k k^2 \\ \sum_{k=0}^{n} {}^{n}C_k k & \sum_{k=0}^{n} {}^{n}C_k 3^k \end{bmatrix} = 0$$

holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^{n}C_k}{k+1}$ equals_____

Sol. **6.2**

3. Let |X| denote the number of elements in a set X, Let S = {1,2,3,4,5,6} be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A,B) such that $1 \leq |B| < |A|$, equals_____

Sol. **1523**

4. The value of the integral

$$\int_{0}^{\pi/2} \frac{3\sqrt{\cos\theta}}{(\sqrt{\cos\theta} + \sqrt{\sin\theta})^5} d\theta$$

equals _

Sol. **0.5**

5. Five persons A,B,C,D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distrubuting the hats such that the persons seated in adjacent seats get different coloured hats is_____

Sol. **30.00**

6. The value of

$$\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$$

in the interval $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$ equals ______

Sol. **0**

Section 3

- This section contains **TWO (02)** List -Match sets
- Each List Match set has **TWO (02)** Multiple Choice Questions.
- Each List Match set has two lists. List I and List II
- **List I** has Four entries (I), (II),(III) and (IV) and List II has Six entries (P),(Q)(R),(S),(T)and (U)
- Four options are given in each multiple choice question based on List I and List II and only one of these four options satisfies the condition asked in the multiple choice question.
- Answer to each question will be evaluated according to the following marking scheme.

Full marks +3 If ONLY the option corresponding to the correct combination is chosen
Zero Marks 0 If none of the options is chosen (i.e., the question is unanswered)
Negative marks –1 in all other cases.

Anser the following by appropriately matching the lists based on the information given in the paragraph

1. Let $f(x) = \sin(\pi\cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$X = \{x : f(x) = 0\}$, $Y = \{x : f'(x) = 0\}$,
$Z = \{x : g(x) = 0\}$, $W = \{x : g'(x) = 0\}$,

List -I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

List-I	**List-II**
(I) X	(P) $\supseteq \left\{\frac{\pi}{2},\frac{3\pi}{2},4\pi,7\pi\right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV) W	(S) $\supseteq \left\{\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}\right\}$
	(T) $\supseteq \left\{\frac{\pi}{3},\frac{2\pi}{3},\pi\right\}$
	(U) $\supseteq \left\{\frac{\pi}{6},\frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?
(1) (III), (P), (Q), (U)
(2) (IV), (P), (R), (S)
(3) (III), (R), (U)
(4) (IV), (Q), (T)

Sol. **2**

2. Answer the following appropriately matching the list based on the information given in the paragraph.

Let $f(x) = \sin(\pi\cos x)$ and $g(x)=\cos(2\pi \sin x)$ be two functions for $x > 0$. Define the following sets whose elements are written in the increasing order.

$X = \{x : f(x)=0\}$, $Y = \{x, f(x) =0\}$

$Z = \{x : g(x)=0\}$, $W = \{x : g'(x)=0\}$

List I contains the sets X,Y,Z and W. List II contains some information regarding these sets.

List I		List II	
(I)	X	(P)	$\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithemetic progression
(III)	Z	(R)	NOT an arithemetic progression
(IV)	W	(S)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?

(1) (I), (Q), (U) (2) (II),(Q), (T) (3) (I), (P), (R) (4) (II), (R), (S)

Sol. **2**

3. Answer the following by appropriately matching the list based on the information given in the paragraph.

Let the circles $C_1 : x^2+y^2=9$ and $C_2 : (x-3)^2+(y-4)^2=16$, intersect at the points X and Y.

Suppose that another circle $C_3 : (x-h)^2+(y-k)^2=r^2$ satisfies the following conditions

(i) centre of C_3 is collinear with the centres of C_1 and C_2.

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expressions given in the List I whose values are given in List II below :

List I		List II	
(I)	$2h + k$	(P)	6
(II)	$\frac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\frac{\text{Area of triangle MZN}}{\text{Area of trangle ZMW}}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

Which of the following is the only CORRECT combination ?

(1) (II), (T) (2) (I), (U) (3) (I), (S) (4) (II), (Q)

Sol. **4**

4. Answer the following by appropriately matching the list based on the information given in the paragraph.

Let the circles $C_1 : x^2+y^2=9$ and $C_2 : (x-3)^2+(y-4)^2=16$, intersect at the points X and Y.

Suppose that another circle $C_3 : (x-h)^2+(y-k)^2=r^2$ satisfies the following conditions

(i) centre of C_3 is collinear with the centres of C_1 and C_2.

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expressions given in the List I whose values are given in List II below :

	List I		**List II**
(I)	$2h + k$	(P)	6
(II)	$\frac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\frac{\text{Area of triangle MZN}}{\text{Area of trangle ZMW}}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

Which of the following is the only CORRECT combination ?

(1) (IV), (S) (2) (IV), (U) (3) (I), (P) (4) (III), (R)

Sol. **1**