MATHS [JEE ADVANCED - 2019] PAPER - 1

SECTION -1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options **ONLY ONE** of these four options is correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +3 If ONLY the correct option is chosen.
Zero Marks : 0 If none of the options is choosen (i.e. the question is unanswered)
Negative marks : –1 In all other cases

1. A line $y = mx + 1$ intersects the circle $(x - 3)^2 + (y + 2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x - coordinate $\frac{-3}{5}$, then which one of the following options is correct ?

(1) $-3 \le mv < -1$ (2) $6 \le m < 8$ (3) $4 \le m < 6$ (4) $2 \le m < 4$

Sol. 4

2. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If
α^* is the minimum of set $\{\alpha(\theta): \theta \in [0,2\pi)\}$ and
β^* is the minimum of the set $\{\beta(\theta): \theta \in [0,2\pi)\}$
then the value of $\alpha^* + \beta^*$ is

(1) $\frac{-29}{16}$ (2) $-\frac{37}{16}$ (3) $-\frac{17}{16}$ (4) $-\frac{31}{16}$

Sol. 1

3. let S be the set of all complex numbers z satsfying $|z - 2 + i| \ge \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) $-\frac{\pi}{2}$

Sol. 4

4. The area of region $\{(x,y) : xy \le 8, 1 \le y \le x^2\}$ is

(1) $16\log_e 2 - \frac{14}{3}$ (2) $8\log_e 2 - \frac{7}{3}$ (3) $8\log_e 2 - \frac{14}{3}$ (4) $16\log_e 2 - 6$

Sol. 1

SECTION -2 (Maximum Marks : 12)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.

  Full marks : +4 If only (all) the correct option(s) is (are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen and both of which are correct
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If two or more options is chosen (i.e. the question is unanswered)
  Negative Marks : –1 in all other cases
- For example, in a question, if (A),(B) and (D) are the ONLY three options corresponding to correct answer, then
  choosing ONLY (A), (B) and (D) will get +4 marks
  choosing ONLY (A) and (B) will get +2 marks
  choosing ONLY (A) and (D) will get +2 marks
  choosing ONLY (B) and (D) will get +2 marks
  choosing ONLY (A) will get +1 mark
  choosing ONLY (B) will get +1 mark
  choosing ONLY (D) will get +1 mark
  choosing no option (i.e., the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark

1. Let Γ denotes a curve y = y(x) which is in the first quadrant and let the point (1,0) lie on it. Let the tangent to Γ at a point P intersect the y - axis at Y_P. If PY_P has length 1 for each point P on Γ, then Which of the following options is/are correct ?

(1) $xy' - \sqrt{1-x^2} = 0$ (2) $y = -\log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

(3) $xy' + \sqrt{1-x^2} = 0$ (4) $y = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

Sol. **1,2,3,4**

2. Define the collections $\{E_1, E_2, E_3......\}$ of ellipse and $\{R_1, R_2, R_3 .....\}$ of rectangles as follows :

$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$;

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1;

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1}, n > 1;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n , n > 1

Then which of the following options is/are correct?

(1)The eccentricities of E_{18} and E_{19} are NOT equal

(2) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

(3) $\sum_{n=1}^{N}(\text{area of } R_n) < 24$, for each positive integer N

(4) The length of latus rectum of E_9 is $\frac{1}{6}$

Sol. **3,4**

3. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj $M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct ?

(1) det $(\text{adj}M^2) = 81$

(2) $a + b = 3$

(3) $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

(4) if $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

Sol. **2,3,4**

4. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integer n, define

$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \geq 1$

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$

Then which of the following options is/are correct ?

(1) $a_1 + a_2 + a_3 + \ldots + a_n = a_{n+2} - 1$ for all $n \geq 1$

(2) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

(3) $\sum_{n=1}^{\infty}\frac{b_n}{10^n} = \frac{8}{89}$

(4) $\sum_{n=1}^{\infty}\frac{a_n}{10^n} = \frac{10}{89}$

Sol. **1,2,4**

5. Let $f : R \to R$ be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\ x^2 - x + 1, & 0 \leq x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \leq x < 3 \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \geq 3 \end{cases}$$

Then which of the following options is /are correct ?

(1) f is increasing on $(-\infty, 0)$

(2) f is onto

(3) f' has a local maximum at x = 1

(4) f' is NOT differentiable at x = 1

Sol. **2,3,4**

6. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct ?

(1) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(2) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(3) Probability that the selected bag is B_3, given that chosen ball is green, equals $\frac{5}{13}$

(4) Probability that the chosen ball is green equals $\frac{39}{80}$

Sol. **1,4**

7. In a non-right angled triangle ΔPQR, let p,q,r denote the lengths of the sides opposite to the angles at P,Q,R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, q=1, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct?

(1) Length of RS = $\frac{\sqrt{7}}{2}$

(2) Length of OE = $\frac{1}{6}$

(3) Radius of incircle ΔPQR = $\frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

(4) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

Sol. **1,2,3**

8. Let L_1 and L_2 denote the lines

$$\vec{r} = \hat{i} + \lambda(-\hat{i} + +2\hat{j} + 2\hat{k}), \lambda \in R \text{ and}$$

$$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in R$$

respectively, If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?

(1) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$

(2) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$

(3) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$

(4) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k})$ $t \in R$

Sol. **1,2**

Section - 3

- This section contains SIX (06) qeustions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/roundoff the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme;
  Full Marks : +3 If ONLY the correct numerical value is entered
  Zero Marks : 0 in all other cases.

1. Three lines are given by

$$\vec{r} = \lambda\hat{i},\ \lambda \in R$$

$$\vec{r} = \mu(\hat{i} + \hat{j}),\ \mu \in R$$

$$\vec{r} = v(\hat{i} + \hat{j} + \hat{k}),\ v \in R$$

Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then value of $(6\Delta)^2$ equals ___________.

Sol. **0.75**

2. Let S be the sample space of all 3 × 3 matrices with entries from the set {0,1}, Let the events E_1 and E_2 be given by

E_1 = {A ∈ S : det A = 0} and

E_2 = {A ∈ S : sum of entries of A is 7}

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals

Sol. **0.5**

3. Let $\omega \neq 1$ be a cube root of unit. Then the minimum of the set

$\{|a + b\omega = c\omega^2|^2$: a, b, c distinct non-zero integers} equals ___________.

Sol. **3**

4. Let AP(a; d) denote the set of all the terms of an infinite arithmetic progression with first term α and common difference d > 0, If

$AP(1;3) \cap AP(2;5) \cap AP(3;7) = AP(a;d)$ then a + d equals ___________.

Sol. **157**

5. If $I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ then $27I^2$ equals ___________.

Sol. **4**

6. Let the point B be the relfection of the point A(2,3) with respect to the line 8x – 6y – 23 = 0. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B resepectively.Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is ___________.

Sol. **10**