Learning Temple
IIT/NEET ACADEMY

7th January 2020 _ SHIFT - II

MATHEMATICS

Learning Temple
IIT/NEET ACADEMY

1. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$, is:

α का वह मान, जिसके लिए $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$, है, है:

(1) $\log_e 2$ (2) $\log_e \sqrt{2}$ (3) $\log_e\left(\frac{4}{3}\right)$ (4) $\log_e\left(\frac{3}{2}\right)$

Sol. **1**

$4\alpha\left\{\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right\} = 5$

$\Rightarrow 4\alpha\left\{\left(\frac{e^{\alpha x}}{\alpha}\right)_{-1}^{0} + \left|\frac{e^{-\alpha x}}{-\alpha}\right|_{0}^{2}\right\} = 5$

$\Rightarrow 4\alpha\left\{\left(\frac{1-e^{-\alpha}}{\alpha}\right) - \left(\frac{e^{-2\alpha}-1}{\alpha}\right)\right\} = 5$

$\Rightarrow 4(2 - e^{-\alpha} - e^{-2\alpha}) = 5$ Put $e^{-\alpha} = t$

$\Rightarrow 4t^2 + 4t - 3 = 0$

$\Rightarrow (2t + 3)(2t - 1) = 0$

$\Rightarrow e^{-\alpha} = \frac{1}{2}$

$\Rightarrow \alpha = \ln 2$

2. The number of ordered pairs (r, k) for which $6.{}^{35}C_r = (k^2 - 3).\ {}^{36}C_{r+1}$, where k is an integer, is:
क्रमित युग्मों (r, k), जिनके लिए $6.{}^{35}C_r = (k^2 - 3).\ {}^{36}C_{r+1}$, जहाँ k एक पूर्णांक है, की संख्या है :
(1) 6 (2) 3 (3) 4 (4) 2

Sol. **3**

$\frac{36}{r+1} \times {}^{35}C_r (k^2 - 3) = {}^{35}C_r$

$k^2 - 3 = \frac{r+1}{6} \Rightarrow k^2 = 3 + \frac{r+1}{6}$

r can be 5, 35
for r = 5, k = ± 2
r = 35, k = ± 3
Hence number of order pair = 4

3. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to:

एक कार्यशाला में पाँच मशीनें है तथा उनमे से एक दिन किसी एक के खराब होने की प्रायिकता $\frac{1}{4}$ है। यदि किसी एक दिन अधिकतम दो मशीन खराब होने की प्रायिकता $\left(\frac{3}{4}\right)^3 k$ है, तो k बराबर है :

(1) $\frac{17}{4}$ (2) $\frac{17}{2}$ (3) 4 (4) $\frac{17}{8}$

Sol. **4**

Required probability = when no. machine has fault + when only one machine has fault + when only two machines have fault.

$$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$$

$$= \frac{243}{1024} + \frac{405}{1024} + \frac{270}{1024} = \frac{918}{1024} = \frac{459}{512} = \frac{27\times17}{64\times8}$$

$$= \left(\frac{3}{4}\right)^3 \times k = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$$

$$\therefore k = \frac{17}{8}$$

4. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is:

यदि किसी $a \in R$ के लिए दीर्घवत्त $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ की एक स्पर्श रेखा $3x + 4y = 12\sqrt{2}$ है, तो दीर्घवत्त की नाभियों के बीच की दूरी है :

(1) $2\sqrt{5}$ (2) $2\sqrt{7}$

(3) $2\sqrt{2}$ (4) 4

Sol. **2**

$3x + 4y = 12\sqrt{2}$

$\Rightarrow 4y = -3x + 12\sqrt{2}$

$\Rightarrow y = -\frac{3}{4}x + 3\sqrt{2}$

condition of tangency $c^2 = a^2m^2 + b^2$

$18 = a^2.\frac{9}{16} + 9$

$a^2.\frac{9}{16} = 9$

$a^2 = 16$

$a = 4$

$$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{9}{16}} = \sqrt{\frac{7}{16}}$$

$\therefore$ ae = $\sqrt{\frac{7}{16}}.4 = \sqrt{7}$

$\therefore$ focus are $(\pm\sqrt{7}, 0)$

$\therefore$ distance between foci = $2\sqrt{7}$

5. The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots\ldots + x^{10}$ is:
व्यंजक $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots\ldots + x^{10}$ में x^7 का गुणांक है :
(1) 420 (2) 210 (3) 330 (4) 120

Sol. 3

$$\frac{(1+x)^{10}\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)}$$

$$\frac{(1+x)^{10}\,|\,(1+x)^{11}-x^{11}\,|}{(1+x)^{11}\times\frac{1}{(1+x)}}$$

$= (1+x)^{11} - x^{11}$

coefficient of x^7 is ${}^{11}C_7 = {}^{11}C_4 = 330$

6. Let $a_1, a_2, a_3, \ldots\ldots$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to:

माना $a_1, a_2, a_3, \ldots\ldots$ गुणोत्तर श्रेढ़ी इस प्रकार है कि $a_1 < 0$, $a_1 + a_2 = 4$ तथा $a_3 + a_4 = 16$. यदि $\sum_{i=1}^{9} a_i = 4\lambda$ है, तो λ बराबर है :

(1) $\frac{511}{3}$ (2) -171 (3) 171 (4) -513

Sol. 2

$a_1 + a_2 = 4 \Rightarrow a_1 + a_1r = 4$ ....(i)
$a_3 + a_4 = 16 \Rightarrow a_1r^2 + a_1r^2 = 16$ ...(ii)

$\frac{1}{r^2} + \frac{1}{4} \Rightarrow r^2 = 4$

$r = \pm 2$

$r = 2,\ a_1(1 + 2) = 4 \Rightarrow a_1 = \frac{4}{3}$

$r = -2,\ a_1(1 - 2) = 4 \Rightarrow a_1 = -4$

$$\sum_{i=1}^{a} a_1 = \frac{a_1\left(r^q-1\right)}{r-1} = \frac{(-4)\left((-2)^9-1\right)}{-2-1} = \frac{4}{3}(-513) = 4\lambda$$

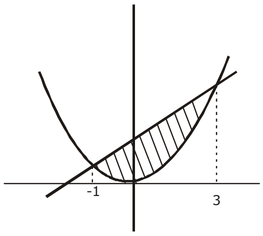
$\lambda = -171$

7. The area (in sq. units) of the region $\{(x, y) \in R^2 4x^2 \le y \le 8x + 12\}$ is :
क्षेत्र $\{(x, y) \in R^2 4x^2 \le y \le 8x + 12\}$ का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{125}{3}$ (2) $\frac{124}{3}$ (3) $\frac{128}{3}$ (4) $\frac{127}{3}$

Sol. **3**

$4x^2 = y$
$y = 8x + 12$
$4x^2 = 8x + 12$
$x^2 - 3x + x - 3 = 0$
$(x + 1)(x - 3) = 0$



$$A = \int_{-1}^{3} (8x + 12 - 4x^2)dx$$

$$A = \left.\frac{8x^2}{2} + 12x - \frac{4x^3}{3}\right|_{-1}^{3} = (4(9) + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right) = 36 + 8 - \frac{4}{3}$$

$$= 44 - \frac{4}{3} = \frac{132 - 4}{3} = \frac{128}{3}$$

8. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, then the ordered pair, $(\lambda, \vec{d})$ is equal to:
माना $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ तीन मात्रक (unit) सदिश इस प्रकार है कि $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. यदि $\lambda = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ तथा $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, तो क्रमित युग्म, $(\lambda, \vec{d})$ बराबर है :

(1) $\left(-\frac{3}{2}, 3\vec{c} \times \vec{b}\right)$ (2) $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$ (3) $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$ (4) $\left(\frac{3}{2}, 3\vec{b} \times \vec{c}\right)$

Sol. **3**

$|\vec{a} + \vec{b} + \vec{c}|^2 = 0$

$3 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow \lambda = \frac{-3}{2}$

$\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a}$

$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$

$\vec{d} = 3(\vec{a} \times \vec{b})$

9. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true?

माना α तथा β समीकरण $x^2 - x - 1 = 0$ के मूल हैं। यदि $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, तो निम्न में से कौन सा एक कथन सत्य नही है ?

(1) $p_5 = p_2 . p_3$ (2) $p_3 = p_5 - p_4$ (3) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$ (4) $p_5 = 11$

Sol. **1**

$\alpha^5 = 5\alpha + 3$
$\beta^5 = 5\beta + 3$
$P_5 = 5(\alpha + \beta) + 6$
$= 5(1) + 6$
$P_5 = 11$ and $P_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$
$P_2 = 3$ and $P_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1 = 2(1) + 2 = 4$
$P_2 \times P_3 = 12$ and $P_5 = 11$
$\Rightarrow P_5 \neq P_2 \times P_3$

10. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where i,j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is:

माना $A = [a_{ij}]$ तथा $B = [b_{ij}]$, 3×3 के दो वास्तविक आव्यूह इस प्रकार है कि $b_{ij} = (3)^{(i+j-2)}a_{ji}$, जहाँ i,j = 1, 2, 3. यदि B का सारणिक 81 है, तो A का सारणिक है :

(1) 1/3 (2) 3 (3) 1/81 (4) 1/9

Sol. **4**

$$|B| = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0 a_{11} & 3^1 a_{11} & 3^2 a_{13} \\ 3^1 a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix}$$

$$\Rightarrow 81 = 3^3.3.3^2|A| \Rightarrow |A| = \frac{1}{9}$$

11. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is:

(1) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$ (2) If $A \subseteq C$, then $B \subset A$ or $D \subset B$
(3) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$ (4) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

माना A, B, C तथा D चार अरिक्त समुच्चय हैं। तो कथन "यदि $A \subseteq B$ तथा $B \subseteq D$, तो $A \subseteq C$" का प्रतिधनात्मक कथन है :–

(1) यदि $A \not\subseteq C$, तो $A \not\subseteq B$ अथवा $B \not\subseteq D$ (2) यदि $A \subseteq C$, तो $B \subset A$ अथवा $D \subset B$
(3) यदि $A \not\subseteq C$, तो $A \not\subseteq B$ तथा $B \subseteq D$ (4) यदि $A \not\subseteq C$, तो $A \subseteq B$ तथा $B \subseteq D$

Sol. **1**

Let $P = A \subseteq B$, $Q = B \subseteq D$, $R = A \subseteq C$
$(P \wedge Q) \rightarrow R$
contrapositive is $\sim R \rightarrow \sim (P \wedge Q)$
$\sim R \rightarrow \sim P \vee \sim Q$

12. Let y = y(x) be the solution curve of the differential equation, $(y^2 - x)\dfrac{dy}{dx} = 1$, satisfying y(0) = 1. This curve intersects the x-axis at a point whose abscissa is:

माना अवकल समीकरण $(y^2 - x)\dfrac{dy}{dx} = 1$ का हल वक्र y = y(x), y(0)=1 को सन्तुष्ट करता है। यह वक्र x-अक्ष को जिस बिन्दु पर काटता है उसका भुज है।

(1) 2 + e (2) 2 - e (3) -e (4) 2

Sol. **2**

$$\frac{dy}{dx} + x = y^2$$

$$\text{I.F.} = e^{\int 1 dy} = e^y$$

$$x.e^y = \int y^2.e^y.dy$$

$$= y^2.e^y - \int 2y.e^y.dy$$

$$\Rightarrow y^2e^y - 2(y.e^y - e^y) + c$$

$$x.e^y = y^2e^y - 2ye^y + 2e^y + c$$

$$x = y^2 - 2y + 2 + c.e^{-y}$$

x = 0, y = 1

$$0 = 1 - 2 + 2 + \frac{c}{e}$$

$$c = -e$$

$$y = 0,\ x = 0 - 0 + 2 + (-e)(e^{-0})$$

$$x = 2 - e$$

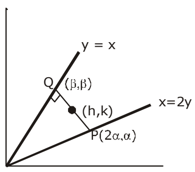
13. The locus of the mid-points of the perpendiculars drawn from points on the line, x = 2y to the line x = y is:

रेखा x = 2y के बिन्दुओं से रेखा x = y पर डाले गये लम्बों के मध्य बिन्दुओं का बिन्दुपथ है :

(1) 7x - 5y = 0 (2) 3x - 2y = 0 (3) 5x - 7y = 0 (4)2x - 3y = 0

Sol. **3**

$$\text{Slope of PQ} = \frac{k-\alpha}{h-2\alpha} = -1$$



$$\Rightarrow k - \alpha = -h + 2\alpha$$

$$\Rightarrow \alpha = \frac{h+k}{3} \quad .....(1)$$

Also $2h = 2\alpha + \beta$

$2k = \alpha + \beta$

$2h = \alpha + 2k$

$\Rightarrow \alpha = 2h - 2k$ .....(2)

from (1) & (2)

$$\frac{h+k}{3} = 2(h-k)$$

So locus is $6x - 6y = x + y \Rightarrow 5x = 7y$

14. If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ... is: (102)m, then m is equal to
यदि श्रेणी 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ... के प्रथम 40 पदों का योगफल (102)m है, तो m बराबर है
(1) 10 (2) 20 (3) 25 (4) 5

Sol. **2**

$S = \underbrace{3+4}+\underbrace{8+9}+\underbrace{13+14}+\underbrace{18+19}......40$ term

S = 7 + 17 + 27 + 37 + 47 + ...20 term

$S_{40} = \frac{20}{2}[2 \times 7 + (19)\,10] = 10[14+190] = 10[2040] = (102)\,(20)$

$\Rightarrow$ m = 20

15. The value of c in the lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is:
फलन $f(x) = x^3 - 4x^2 + 8x + 11, x \in [0, 1]$ के लिए लग्रांज मध्यमान प्रमेय में c का मान है:

(1) $\frac{\sqrt{7}-2}{3}$ (2) $\frac{4-\sqrt{7}}{3}$ (3) $\frac{4-\sqrt{5}}{3}$ (4) $\frac{2}{3}$

Sol. **2**

f(x) is a polynomial function
$\therefore$ it is continuous and differentiable in [0,1]
Here f(0) = 11, f(1) = 1 - 4 + 8 + 11 = 16
$f'(x) = 3x^2 - 8x + 8$

$\therefore f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{16-11}{1} = 3c^2 - 8c + 8$

$\Rightarrow 3c^2 - 8c + 3 = 0$

$C = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$

$\therefore \quad C = \frac{4-\sqrt{7}}{3} \in (0, 1)$

16. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to:
माना मूल बिन्दु से वत्त $x^2 + y^2 - 8x - 4y + 16 = 0$ पर खींची गई स्पर्श रेखायें इसे बिन्दुओं A तथा B पर स्पर्श करती है। तो $(AB)^2$ बराबर है :

(1) $\frac{56}{5}$ (2) $\frac{52}{5}$ (3) $\frac{64}{5}$ (4) $\frac{32}{5}$

Sol. **3**

$L = \sqrt{S_1} = \sqrt{16} = 4$

$R = \sqrt{16+4-16} = 2$

Length of chord of contact = $\frac{2LR}{\sqrt{L^2+R^2}} = \frac{2 \times 4 \times 2}{\sqrt{16+4}} = \frac{16}{\sqrt{20}}$

Square of length of chord of contact $= \frac{64}{5}$

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17. Let y = y(x) be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at x = $\frac{1}{2}$, is equal to:

माना x का एक फलन y = y(x), जो $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ को सतुष्ट करता है जहाँ k एक अचर है तथा $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. तो x = $\frac{1}{2}$ पर $\frac{dy}{dx}$ बराबर है :

(1) $-\frac{\sqrt{5}}{2}$ (2) $\frac{2}{\sqrt{5}}$ (3) $\frac{\sqrt{5}}{2}$ (4) $-\frac{\sqrt{5}}{4}$

Sol. **1**

$$x = \frac{1}{2}, y + \frac{-1}{4} \Rightarrow \quad xy = \frac{-1}{8}$$

$$y.\frac{1.(2x)}{2\sqrt{-x^2}} + y'.\sqrt{1-x^2} = -\left\{1.\sqrt{1-y^2} + \frac{x.(-2y)}{2\sqrt{1-y^2}}y'\right\}$$

$$-\frac{xy}{\sqrt{1-x^2}} + y'\sqrt{1-x^2} = -\sqrt{1-y^2} + \frac{xy.y'}{\sqrt{1-y^2}}$$

$$y'\left(\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}\right) = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$$

$$y'\left(\frac{\sqrt{3}}{2} + \frac{1}{8.\frac{\sqrt{15}}{4}}\right) = \frac{-1}{8.\sqrt{\frac{3}{2}}} - \frac{\sqrt{15}}{4}$$

$$y'\left(\frac{\sqrt{45}+1}{2\sqrt{15}}\right) = -\frac{\left(1+\sqrt{45}\right)}{4\sqrt{3}}$$

$$y' = -\frac{\sqrt{5}}{2}$$

18. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ is equal to:

$(0, 2\pi) - \{\pi\}$ में समीकरण $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$ को सन्तुष्ट करने वाले θ के न्यूनतम तथा अधिकतम मान क्रमश θ_1 तथा θ_2 हैं, तो $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ बराबर है :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{3} + \frac{1}{6}$ (3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{9}$

Sol. **1**

$$2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$$

$$\frac{2\cos^2\theta}{\sin^2\theta} - \frac{5}{\sin\theta} + 4 = 0$$

$$2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0, \sin\theta \neq 0$$

$$2\sin^2\theta - 5\sin\theta + 2 = 0$$

$$(2\sin\theta - 1)(\sin\theta - 2) = 0$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\therefore \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6} \frac{1+\cos 6\theta}{2} d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 6\theta}{6}\right]_{\pi/6}^{5\pi/6} = \frac{1}{2}\left[\frac{5\pi}{6} - \frac{\pi}{6} + \frac{1}{6}(0-0)\right] = \frac{1}{2}.\frac{4\pi}{6} = \frac{\pi}{3}$$

19. If $\frac{3 + i\sin\theta}{4 - i\cos\theta}$, $\theta \in [0, 2\theta]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is:

यदि $\frac{3 + i\sin\theta}{4 - i\cos\theta}$, $\theta \in [0, 2\theta]$, एक वास्तविक संख्या है, तो सम $\sin\theta + i\cos\theta$ का एक कोणांक (argument) है :

(1) $-\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$ (3) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$ (4) $\tan^{-1}\left(\frac{4}{3}\right)$

Sol. **2**

$$z = \frac{(3 + i\sin\theta)}{(4 - i\cos\theta)} \times \frac{(4 + i\cos\theta)}{4 + i\cos\theta}$$

as z is purely real $\Rightarrow$ 3 cos θ + 4 sin θ = 0 $\Rightarrow$ tan θ $= -\frac{3}{4}$

arg(sinθ + icosθ) = π + $\tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$ = θ + $\tan^{-1}\left(-\frac{4}{3}\right)$ = π - $\tan^{-1}\left(\frac{4}{3}\right)$

20. Let $f(x)$ be a polynomial of degree 5 such that x = ±1 are its critical points. If $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

(1) x = 1 is a point of minima and x = -1 is a point of maxima of f.
(2) x = 1 is a point of maxima and x = -1 is a point of minimum of f.
(3) $f(1) - 4f(-1) = 4$.
(4) f is an odd function.

माना 5 घात के एक बहुपद $f(x)$ के क्रांतिक बिन्दु x = ±1 है। यदि $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)$ = 4 है, तो निम्न में से कौन सा एक सत्य नही है ?

(1) f का एक उच्चिष्ठ बिन्दुं x = 1 है तथा एक निम्ननिष्ठ बिन्दु x = -1 है।
(2) f का एक निम्ननिष्ठ बिन्दु x = 1 है तथा एक उच्चिष्ठ बिन्दु x = -1 है।
(3) $f(1) - 4f(-1) = 4$.
(4) f एक विषम फलन है।

Sol. **1**

$f(x) = ax^5 + bx^4 + cx^3$

$\lim_{x\to 0}\left(2+\frac{ax^5+bx^4+cx^3}{x^3}\right) = 4 \Rightarrow 2 + c = 4 \Rightarrow c = 2$

$f'(x) = 5ax^4 + 4bx^3 + 6x^2$
$= x^2(5ax^2 + 4bx + 6)$
$f'(1) = 0 \Rightarrow 5a + 4b + 6 = 0$
$f'(-1) = 0 \Rightarrow 5a - 4b + 6 = 0$
$b = 0$

$a = -\frac{6}{5}$

$f(x) = \frac{-6}{5}x^5 + 2x^3$

$f'(x) = -6x^4 + 6x^2$
$= 6x^2(-x^2 + 1)$
$= -6x^2(x + 1)(x - 1)$

-1 + 1-
1- 1

minimal at x = -1
maxima at x = 1

21. Let X = {$n \in N : 1 \le n \le 50$}. If
A = {$n \in X$: n is a multiple of 2} and
B = {$n \in X$: n is a multiple of 7}, then the number of elements in the smallest subset of X containing both A and B is ________.
माना X = {$n \in N : 1 \le n \le 50$}. यदि
A = {$n \in X$: n, 2 का एक गुणज है} तथा
B = {$n \in X$: n ,7 का एक गुणज है}, तो X के सबसे छोटे उपसमुच्चय, जिसमें A तथा B दोनों है, में अवयवों की संख्या है________.

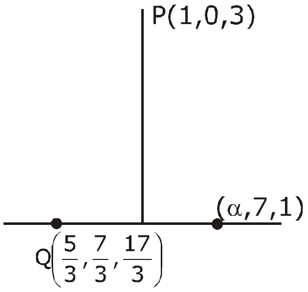
Sol. **29**

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 25 + 7 - 3 = 29$

22. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to______.

. यदि (α, 7, 1) से जाने वाली एक रेखा पर बिन्दु (1, 0, 3) से डाले गये लम्ब का पाद $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ है, तो α बराबर है______.

Sol. **4**
Since PQ is perpendicular to L, therefore



$$\left(1-\frac{5}{3}\right)\left(\alpha-\frac{5}{3}\right) + \left(\frac{-7}{3}\right)\left(7-\frac{7}{3}\right) + \left(3-\frac{17}{3}\right)\left(1-\frac{17}{3}\right) = 0$$

$$\Rightarrow \frac{-2\alpha}{3} + \frac{10}{9} - \frac{98}{9} + \frac{112}{9} = 0$$

$$\Rightarrow \frac{2\alpha}{3} = \frac{24}{9} \Rightarrow \alpha = 4$$

23. If the system of linear equations,
x + y + z = 6
x + 2y + 3z = 10
3x + 2y + λz = μ
has more than two solutions, then $\mu - \lambda^2$ is equal to ______.
यदि रैखिक समीकरण निकाय
x + y + z = 6
x + 2y + 3z = 10
3x + 2y + λz = μ
के दो से अधिक हल हैं, तो $\mu - \lambda^2$ बराबर है

sol. **13**

$x + y + z = 6$ ....(1)
$x + 2y + 3z = 10$ ...(2)
$3x + 2y + \lambda z = \mu$ ....(3)
from (1) and (2)
if $z = 0 \Rightarrow x + y = 6$ and $x + 2y = 10$
$\Rightarrow y = 4, x = 2$
(2,4,0)
if $y = 0 \Rightarrow x + z = 6$ and $x + 3z = 10$
$\Rightarrow z = 2$ and $x = 4$
(4,0,2)
So, $3x + 2y + \lambda z = \mu$
must pass through (2,4,0) and (4,0,2)
so $6 + 8 = \mu \Rightarrow \mu = 14$
and $12 + 2\lambda = \mu$
$12 + 2\lambda = 14 \Rightarrow \lambda = 1$
so $\mu - \lambda^2 = 14 - 1$
$= 13$

24. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x.y is equal to ________.
यदि आठ संख्याओं 3, 7, 9, 12, 13, 20, x तथा y के माध्य तथा प्रसरण क्रमशः 10 तथा 25 हैं, तो xy बराबर है ________.

sol. **52**

mean = $\bar{x} = \frac{3+7+9+12+13+20+x+y}{8} = 10 \Rightarrow x + y = 16$ ...(i)

variance $\sigma^2 = \frac{\sum (x_i)^2}{8} - (\bar{x})^2 = 25$

$$\frac{9+49+81+144+169+400+x^2+y^2}{8} - 100 = 25$$

$\Rightarrow x^2 + y^2 = 148$ ...(ii)
$(x+y)^2 = (16)^2 \Rightarrow x^2 + y^2 + 2xy = 256 \Rightarrow xy = 52$

25. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by $f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \neq 0 \\ k & \text{when } x = 0 \end{cases}$ is continuous, then k is equal to________.

यदि $\left(-\frac{1}{3}, \frac{1}{3}\right)$ में $f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \neq 0 \\ k & \text{when } x = 0 \end{cases}$ द्वारा परिभाषित फलन, f संतत है, तो k बराबर है________।

Sol. **5**

$$\lim_{x\to 0} f(x) \lim_{x\to 0}\left(\frac{1}{x}\ell n\left(\frac{1+3x}{1-2x}\right)\right) = \lim_{x\to 0}\left(\frac{\ell n(1+3x)}{x} - \frac{\ell n(1-2x)}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{3\ell n(1+3x)}{3x} - \frac{2\ell n(1-2x)}{-2x}\right) = 3 + 2 = 5$$

$\therefore$ f(x) will be continuous. if f(0) = $\lim_{x\to 0} f(x)$