Learning Temple

IIT/NEET ACADEMY

26th Feb. 2021 | Shift - 1
MATHEMATICS

1. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

(1) 77

(2) 42

(3) 35

(4) 82

Ans. **(1)**

Sol. CASE-I: 1, 1, 1, 1, 1, 2, 3

WAYS = $\frac{7!}{5!}$ = 42

CASE-II: 1, 1, 1, 1, 2, 2, 2

WAYS = $\frac{7!}{4!.3!} = 35$

TOTAL WAYS = 42 + 35 = 77

2. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is:

(1) $\frac{10!}{\sqrt{3}(5!)^2}$

(2) $\frac{2.10!}{3(5!)^2}$

(3) $\frac{10!}{3(5!)^2}$

(4) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

Ans. **(4)**

Sol. $T_{r+1} = {}^{10}C_r (tx^{1/5})^{10-r} \left[\frac{(1-x)^{1/10}}{t}\right]^r$

$= {}^{10}C_r t^{(10-2r)} \times x^{\frac{10-r}{5}} \times (1-x)^{\frac{r}{10}}$

$\Rightarrow$ 10 −2r = 0 $\Rightarrow$ r = 5

$T_6 = {}^{10}C_5 x\sqrt{1-x}$

$\frac{dT_6}{dx} = {}^{10}C_5 \left[\sqrt{1-x} - \frac{x}{2\sqrt{1-x}}\right] = 0$

= 1 − x = x/2 $\Rightarrow$ 3x = 2

$\Rightarrow$ x = 2/3

$T_6|_{max} = \frac{10!}{5!5!} \times \frac{2}{3\sqrt{3}}$

3. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\le x$, is:

(1) 100 (e−1)
(2) 100e
(3) 100(1-e)
(4) 100 (1 + e)

Ans. **(1)**

Sol. $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx$

$= \int_0^1 e^{\{x\}}dx + \int_1^2 e^{\{x\}}dx + \int_2^3 e^{\{x\}}dx + ... \int_{99}^{100} e^{\{x\}}dx$ ($\because$ {x}= x − [x])

$= e^x |_0^1 + e^{(x-1)} |_1^2 + e^{(x-2)} |_2^3 + ... + e^{(x-99)} |_{99}^{100}$

$= (e-1) + (e-1) + (e-1) + ...(e-1)$

$= 100(e-1)$

4. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours,

then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

(1) 4
(2) 2
(3) 16
(4) 8

Ans. **(1)**

Sol. $\frac{dx}{dt} \propto x$

$\frac{dx}{dt} = \lambda x$

$\int_{1000}^{x} \frac{dx}{x} = \int_{0}^{t} \lambda dt$

$\ln x - \ln 1000 = \lambda t$

$\ln\left(\frac{x}{1000}\right) = \lambda t$

Put t = 2, x = 1200

$\ln\left(\frac{12}{10}\right) = 2\lambda \Rightarrow \lambda = \frac{1}{2}\ln\frac{6}{5}$

Now $\ln\left(\frac{x}{1000}\right) = \frac{t}{2}\ln\left(\frac{6}{5}\right)$

$x = 1000e^{\frac{t}{2}\ln\left(\frac{6}{5}\right)}$

$x = 2000$ at $t = \frac{k}{\ln\left(\frac{6}{5}\right)}$

$\Rightarrow 2000 = 1000\, e^{\frac{k}{2\ln(6/5)} \times \ln(6/5)}$

$\Rightarrow 2 = e^{k/2}$

$\Rightarrow \ln 2 = \frac{k}{2}$

$\Rightarrow \frac{k}{\ln 2} = 2$

$\Rightarrow \left(\frac{k}{\ln 2}\right)^2 = 4$

5. If $\vec{a}$ & $\vec{b}$ are perpendicular vactors, then

$\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)$ is equal to

(1) $\frac{1}{2}|\vec{a}|^4 \vec{b}$

(2) $\vec{a} \times \vec{b}$

(3) $|\vec{a}|^4 \vec{b}$

(4) $\vec{0}$

Ans. **(3)**

Sol. $\vec{a}\times\left(\vec{a}\times\left((\vec{a}.\vec{b})\vec{a}-|\vec{a}|^2\vec{b}\right)\right)$

$\vec{a}\times(-|\vec{a}|^2(\vec{a}\times\vec{b})) = -|\vec{a}|^2((\vec{a}.\vec{b})\vec{a}-|\vec{a}|^2\vec{b})$

$= -(\vec{a}.\vec{b})\vec{a}|\vec{a}|^2+|\vec{a}|^4\vec{b}$

$= |\vec{a}|^4\vec{b}\quad(\because \vec{a}\cdot\vec{b}=0)$

6. In an increasing, geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to :

(1) 35

(2)30

(3) 26

(4) 32

Ans **(1)**

Sol. $ar + ar^5 = \frac{25}{2}$

$ar^2 \times ar^4 = 25$

$a^2r^6 = 25$

$ar^3 = 5$

$\boxed{a=\frac{5}{r^3}}$ ....(1)

$\frac{5r}{r^3}+\frac{5r^5}{r^3}=\frac{25}{2}$

$\frac{1}{r^2}+r^2=\frac{5}{2}$

Put $r^2 = t$

$\frac{t^2+1}{t}=\frac{5}{2}$

$2t^2 - 5t + 2 = 0$

$2t^2 - 4t - t + 2 = 0$

$(2t - 1)(t - 2) = 0$

$t=\frac{1}{2},2 \quad\Rightarrow \boxed{r^2=\frac{1}{2},2}$

$\boxed{r = \sqrt{2}}$

$= ar^3 + ar^5 + ar^7$

$= ar^3 (1 + r^2 + r^4)$

$= 5[1 + 2 + 4] = 35$

7. Consider the three planes

$P_1 : 3x + 15y + 21z = 9,$

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true?

(1) P_1 and P_3 are parallel.

(2) P_2 and P_3 are parallel.

(3) P_1 and P_2 are parallel.

(4) P_1, P_2 and P_3 all are parallel.

Ans. **(1)**

Sol. $P_1 = x + 5y + 7z = 3$

$P_2 = x - 3y - z = 5$

$P_3 = x + 5y + 7z = 5/2$

$\Rightarrow P_1 \| P_3$

8. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots$ is equal to

(1) $\frac{9}{4}$

(2) $\frac{15}{4}$

(3) $\frac{13}{4}$

(4) $\frac{11}{4}$

Ans. **(3)**

Sol. $s = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + ...$

$\frac{s}{3} = \quad \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + ...\infty$

$\frac{2s}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + ...\infty$

$\frac{2s}{3} = \frac{4}{3} + \frac{5}{3}\left\{\frac{1/3}{1-\frac{1}{3}}\right\} \qquad = \frac{5}{6} + \frac{4}{3} = \frac{13}{6}$

$\boxed{s = \frac{13}{4}}$

9. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) −2

(2) (a+1) (a+2) (a+3)

(3) 0

(4) (a+2) (a+3) (a+4)

Ans. **(1)**

Sol. $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$= \begin{vmatrix} (a+2)a & a+1 & 1 \\ (a+3)(a+1) & a+2 & 1 \\ (a+4)(a+2) & a+3 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$= \begin{vmatrix} a^2+2a & a+1 & 1 \\ 2a+3 & 1 & 0 \\ 4a+8 & 2 & 0 \end{vmatrix}$

$= 6 - 8 = -2$

10. If $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}; 0<x<1,$ then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is:

(1) $\frac{1-y^2}{2y}$

(2) $\frac{1-y^2}{1+y^2}$

(3) $1-y^2$

(4) $\frac{1-y^2}{y\sqrt{y}}$

Ans. (2)

Sol. $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}$

$$\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\sin^{-1}x+\cos^{-1}x}{a+b}=\frac{\pi}{2(a+b)}$$

Now, $\frac{\tan^{-1}y}{c}=\frac{\pi}{2(a+b)}$

$$2\tan^{-1}y=\frac{\pi c}{a+b}$$

$$\Rightarrow \quad \cos\left(\frac{\pi c}{a+b}\right)=\cos\left(2\tan^{-1}y\right)=\frac{1-y^2}{1+y^2}$$

11. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is:

(1) 6

(2) 1

(3) 4

(4)12

Ans. (3)

Sol. Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & ab + bc \\ ab + bc & c^2 + b^2 \end{bmatrix}$$

$= a^2 + 2b^2 + c^2 = 1$

$a = 1, b = 0, c = 0$

$a = 0, b = 0, c = 1$

$a = -1, b = 0, c = 0$

$c = -1, b = 0, a = 0$

12. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a:

(1) Equilateral triangle

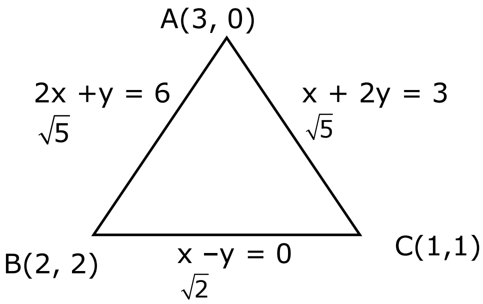
(2) None of the above

(3) Isosceles triangle

(4) Right angled triangle

Ans. **(3)**

Sol.



13. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the point:

(1) (2, 9)

(2) (2,2)

(3) $\left(3, \frac{21}{2}\right)$

(4) (0, 0)

Ans. **(2)**

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Let $f(x) = 2x^3 - 15x^2 + 36x - 19$

$f'(x) = 6x^2 - 30x + 36 = 0$

$x^2 - 5x + 6 = 0$

$\boxed{x = 2, 3}$

$f''(x) = 12x - 30$

$f''(x) < 0$ for $\boxed{x = 2}$

At $\boxed{x = 2}$

$y = 8 - 40 + 72 - 38$

$y = 72 - 70 = 2$

$\Rightarrow (2,2)$

14. Let *f* be any function defined on R and let it satisfy the condition :

$(|f(x) - f(y)| \le |(x-y)^2|, \forall (x, y) \in R$

If f(0) = 1, then:

(1) $f(x) < 0, \ \forall x \in R$

(2) f(x) can take any value in R

(3) $f(x) = 0, \ \forall \ x \in R$

(4) $f(x) > 0, \ \forall \ x \in R$

Ans. (4)

Sol. $|f(x) - f(y)| \le |(x-y)^2|, \forall (x, y) \in R$

$$\left|\frac{f(x) - f(y)}{x - y}\right| \le |x - y|$$

$$\lim_{x \to y}\left|\frac{f(x) - f(y)}{x - y}\right| \le 0$$

$|f'(y)| \le 0 \Rightarrow f'(y) = 0$

$f(y) = C$

$\Rightarrow \boxed{c = 1}$ $\qquad \because \ f(0) = 1$

$\Rightarrow f(x) = 1$

15. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1 + 3^x} dx$ is:

(1) 2π
(2) 4π
(3) $\frac{\pi}{2}$
(4) $\frac{\pi}{4}$

Ans. **(4)**

Sol. Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$

$I = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$

$2I = \int_{-\pi/2}^{\pi/2} \cos^2 x dx$

$I = \int_{0}^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$

16. The value of $\lim_{h\to 0}\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$ is:

(1) $\frac{3}{4}$

(2) $\frac{2}{\sqrt{3}}$

(3) $\frac{4}{3}$

(4) $\frac{2}{3}$

Ans. (3)

Sol. $\lim_{h\to 0} 2\times 2\left\{\frac{\sin\left(\frac{\pi}{6}+h-\frac{\pi}{6}\right)}{2\sqrt{3}h\left(\cos(h+\frac{\pi}{6}\right)}\right\}$

$= \frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = \frac{4}{3}$

17. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

(1) $\frac{15}{2^{12}}$

(2) $\frac{15}{2^{13}}$

(3) $\frac{15}{2^{14}}$

(4) $\frac{15}{2^{8}}$

Ans. (2)

Sol. $p(x=9) = p(x=7)$

$${}^{n}C_9\left(\frac{1}{2}\right)^{n-9}\times\left(\frac{1}{2}\right)^{9} = {}^{n}C_7\left(\frac{1}{2}\right)^{n-7}\times\left(\frac{1}{2}\right)^{7}$$

$${}^{n}C_9\times\left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{2}\times{}^{n}C_7$$

$x + y = n \quad \Rightarrow \; n = 16$

$$p(x=2) = {}^{16}C_2\times\left(\frac{1}{2}\right)^{14}\times\left(\frac{1}{2}\right)^{2}$$

$$= {}^{16}C_2\times\left(\frac{1}{2}\right)^{16} = \frac{15}{2^{13}}$$

18. If (1,5,35), (7,5,5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is:

(1) $-\frac{44}{5}$

(2) $\frac{39}{5}$

(3) $-\frac{39}{5}$

(4) $\frac{44}{5}$

Ans. **(4)**

Sol. Let P(1,5,35), Q(7,5,5), R(1, λ,7), S(2λ,1,2)

$$\left[\vec{PQ}\ \vec{PR}\ \vec{PS}\right] = 0$$

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & -5 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$$\{-33\lambda + 165 - 112\} + 5(\lambda - 5)(2\lambda - 1) = 0$$

$$53 - 33\lambda + 5\{2\lambda^2 - 11\lambda + 5\} = 0$$

$$16\lambda^2 - 88\lambda + 78 = 0$$

$$5\lambda^2 - 44\lambda + 39 = 0 <^{\lambda_1}_{\lambda_2}$$

$$\Rightarrow \lambda_1 + \lambda_2 = 44/5$$

19. Let R = {P,Q)|P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set:

(1) S = {(x,y)|$x^2+y^2=1$}

(2) S = {(x,y)|$x^2+y^2=4$}

(3) S = {(x,y)|$x^2+y^2=\sqrt{2}$ }

(4) S = {(x,y)|$x^2+y^2= 2$}

Ans. **(4)**

Sol. P(a, b) , Q(c, d), PO = QO

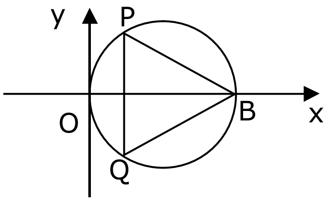
$\Rightarrow a^2 + b^2 = c^2 + d^2$

R(x,y) s = (1, –1) $\Rightarrow$ RO = SO

($\because$ equivalence class)

$x^2 + y^2 = 2$

20. In the circle given below, let OA = 1 unit, OB = 13 unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is:



(1) $26\sqrt{3}$

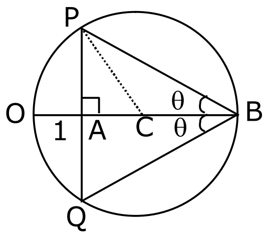
(2) $24\sqrt{2}$

(3) $24\sqrt{3}$

(4) $26\sqrt{2}$

Ans. (3)

Sol.



$OC = \frac{13}{2} = 6.5$

$AC = CO - AO$

$= 6.5 - 1$

$= 5.5$

In $\triangle PAC$

$PA = \sqrt{6.5^2 - 5.5^2}$

$PA = \sqrt{12}$

$\Rightarrow PQ = 2PA = 2\sqrt{12}$

Now, area of $\triangle PQB = \frac{1}{2} \times PQ \times AB$

$= \frac{1}{2} \times 2\sqrt{12} \times 12$

$= 12\sqrt{12}$

$= 24\sqrt{3}$

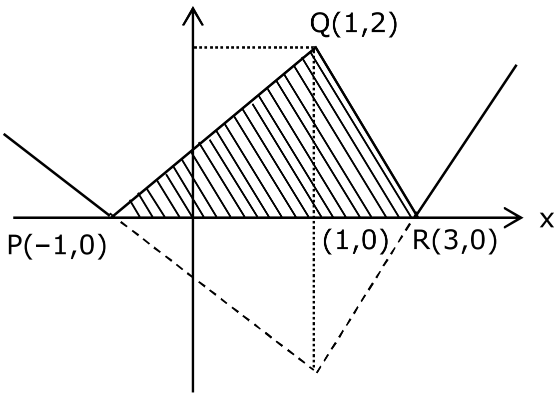
Section-B

1. The area bounded by the lines $y = ||x-1|-2|$ is.......

Ans. **Bonus**

NTA Ans. (8)

Sol.



2. The number of integral values of 'k' for which the equation 3sinx + 4cos x = k + 1 has a solution, $k \in R$ is _______.

Ans. **(11)**

Sol. $3\sin x + 4\cos x = k+1$

$-5 \le k+1 \le 5$

$-6 \le k \le 4$

$\boxed{-6,-5,-4,-3,-2,-1,0,1,2,3,4}$ $\Rightarrow$ 11 integral values

3. Let $m, n \in N$ and gcd (2, n) = 1. If $30\binom{30}{0}+29\binom{30}{1}+....+2\binom{30}{28}+1\binom{30}{29}=n\cdot 2^m$,

then n + m is equal to______.

Ans. **(45)**

Sol. Let $S=\sum_{r=0}^{30}(30-r)\ {}^{30}C_r$

$= 30\sum_{r=0}^{30}{}^{30}C_r - \sum_{r=0}^{30} r\ {}^{30}C_r$

$= 20 \times 2^{30} - \sum_{r=1}^{30} r.\frac{30}{4}.^{29}C_{r-1}$

$= 30 \times 2^{30} - 30.2^{29}$

$= (30\times 2 - 30).2^{29} = 30.2^{29} \Rightarrow 15.2^{30}$

$= n = 15,\ m = 30$

$n + m = 45$

4. If $y = y(x)$ is the solution of the equation $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x, y(0) = 0$; then

$1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2} y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}} y\left(\frac{\pi}{4}\right)$ is equal to ______.

Ans. **(1)**

Sol. $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$

Put $e^{\sin y} = t$

$e^{\sin y} \times \cos y \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} + t \cos x = \cos x$

I.F. $= e^{\int \cos x dx} = e^{\sin x}$

Solution of diff equation:

$t.e^{\sin x} = \int e^{\sin x}.\cos x dx$

$e^{\sin y}.e^{\sin x} = e^{\sin x} + c$

at $x = 0, y = 0$

$1 = 1 + c \qquad \Rightarrow c = 0$

$e^{\sin x + \sin y} = e^{\sin x}$

$\sin x + \sin y = \sin x$

$y = 0$

$\Rightarrow y\left(\frac{\pi}{6}\right) = 0, \quad y\left(\frac{\pi}{3}\right) = 0, \quad y\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow 1 + 0 + 0 + 0 = 1$

5. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is _______.

Ans. (1)

Sol. $\frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$x-1 = (x-3)^2$

$x^2 - 6x + 9 = x - 1$

$x^2 - 7x + 10 = 0$

$x = 2, 5$

X = 2 Not possible as $\log_2(x-3)$ is not defined

$\Rightarrow$ No. of solution = 1

6. If $\sqrt{3}(\cos^2 x) = (\sqrt{3}-1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is ________.

Ans. (1)

Sol. $\sqrt{3}t^2 - \left(\sqrt{3}-1\right)t - 1 = 0$ (where t = cosx)

Now, $t = \frac{\left(\sqrt{3}-1\right) \pm \sqrt{4+2\sqrt{3}}}{2\sqrt{3}}$

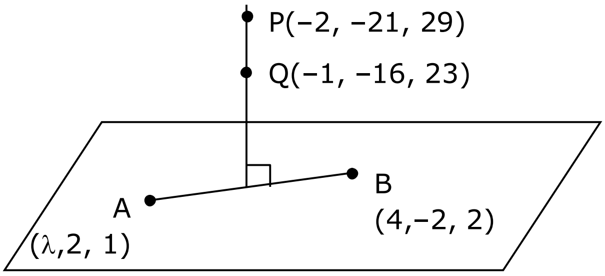
t = cos x = 1 or $-\frac{1}{\sqrt{3}}$ $\rightarrow$ rejected as $x \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow$ cos x = 1

$\Rightarrow$ No. of solution = 1

7. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the point $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to _______.

Ans. **(8)**
Sol.



$\vec{AB} \perp \vec{PQ}$

$\left[(4-\lambda)\hat{i} - 4\hat{j} + \hat{k}\right].\left[+\hat{i} + 5\hat{j} - 6\hat{k}\right] = 0$

$4 - \lambda - 20 - 6 = 0$

$\boxed{\lambda = -22}$

Now , $\frac{\lambda}{11} = -2$

$\Rightarrow \left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$

$\Rightarrow \quad 4 + 8 - 4 = 8$

8. The difference betweeen degree and order of differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is _______.

Ans. **(2)**

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$

$2yy' = a$

$y^2 = 2yy'\left(x + \frac{\sqrt{2yy'}}{2}\right)$

$y = 2y'\left(x + \frac{\sqrt{yy'}}{\sqrt{2}}\right)$

$y - 2xy' = \sqrt{2}y'\sqrt{yy'}$

$\left(y - 2x\frac{dy}{dx}\right)^2 = 2y\left(\frac{dy}{dx}\right)^3$

D = 3 & O = 1

D– O = 3 – 1 = 2

9. The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is______.

Ans. (3)

Sol. Let roots of $x^3 - 2x^2 + 2x - 1 = 0$ are α, β, γ

$(x - 1)(x^2 - x + 1) = 0$

$x = \underset{\alpha}{\underset{\downarrow}{1}}, \; \underset{\beta}{\underset{\downarrow}{-\omega}}, \; \underset{\gamma}{\underset{\downarrow}{-\omega^2}}$

Now $\alpha^{162} + \beta^{162} + \gamma^{162}$

$= 1 + \omega^{162} + (\omega^2)^{162}$

$= 1 + (\omega^3)^{54} + (\omega^3)^{108}$

$= 3$

10. The value of the integeral $\int_0^{\pi} |\sin 2x|\, dx$ is ________

Ans. (2)

Sol. $I = \int_0^{\pi} |\sin 2x|\, dx$

$I = 2\int_0^{\pi/2} |\sin 2x|\, dx = 2\int_0^{\pi/2} \sin 2x\, dx$

$I = 2\left[\frac{-\cos(2x)}{2}\right]_0^{\pi/2} = 2$